Spiral plat avec courbes terminales externe et interne

Déformée élastique en position horizontale

Cas d'une montre bracelet

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.03\,mm \quad ha = 0.15\,mm \quad S = 4.5\times 10^{-3}\,mm^2 \quad TOL := 10^{-12}$

$d2_{sp} = 4.52\,mm \quad d_V := 1.1\cdot mm \quad d_B := 1.312\cdot d1_{sp} \quad p_{sp} = 0.135\,mm \quad n_{sp} := \dfrac{d2_{sp} - d_B}{2\cdot p_{sp}}$

$L := \pi\cdot\dfrac{n_{sp}}{2}\cdot\left(d2_{sp} + d_B\right) \quad L = 10.674\,cm \quad \psi_0 := 2\cdot\pi\cdot n_{sp} \quad \psi_0 = 4.102\times 10^3\,deg$

Position du point de raccordement sur le spiral $\quad \alpha_A := \pi \quad r_A := 0.5\cdot d2_{sp} \quad z_A := r_A\cdot e^{i\cdot\alpha_A}$

Forme initiale du spiral

$a := \dfrac{p_{sp}}{2\cdot\pi} \quad r_s(\alpha) := r_A - a\cdot(\alpha - \alpha_A) \quad x_{0s}(\alpha) := r_s(\alpha)\cdot cos(\alpha) \quad y_{0s}(\alpha) := r_s(\alpha)\cdot sin(\alpha)$

$s(\alpha) := \dfrac{1}{2\cdot a}\cdot\left(r_A^{\,2} - r_s(\alpha)^2\right) \quad s(\alpha) := r_A\cdot(\alpha - \alpha_A) - \dfrac{a}{2}\cdot(\alpha - \alpha_A)^2 \quad s(\psi_0 + \pi) = 10.674\,cm$

Courbe terminale externe

$r_{t1} := 0.8 \quad r_{t1} := racine\left[(2\cdot r_{t1} - 1)^4 - 4\cdot(1 - r_{t1})^4 - \pi^2\cdot r_{t1}^{\,2}\cdot(1 - r_{t1})^2, r_{t1}\right]\cdot r_A \quad r_{t1} = 0.832\,r_A$

$r_{t2} := 2\cdot r_{t1} - r_A \quad r_{t2} = 0.665\,r_A \quad \beta_0 := arctan\left[\dfrac{\pi\cdot r_{t1}}{2\cdot(r_A - r_{t1})}\right] \quad \beta_0 = 82.695\,deg \quad l_t := r_{t2}\cdot\beta_0 + \pi\cdot r_{t1}$

$x_{0t1}(\alpha_t) := -r_A + r_{t1}\cdot(1 + cos(\alpha_t)) \quad y_{0t1}(\alpha_t) := r_{t1}\cdot sin(\alpha_t)$

$x_{0t2}(\beta_t) := r_{t2}\cdot cos(\beta_t) \quad y_{0t2}(\beta_t) := r_{t2}\cdot sin(\beta_t)$

Courbe terminale interne $\quad \alpha_B := mod(\psi_0 + \pi, 2\cdot\pi) \quad \alpha_B = 322.4\,deg \quad r_B := 0.5\cdot d_B$

$\beta := 121\cdot deg \quad \beta'_0 := racine\left[\beta\cdot\left(\sqrt{2}\cdot sin(\beta) - 1\right) + sin(\beta)\cdot cos(\beta), \beta\right] \quad \beta'_0 = 121.21\,deg$

$r_{t'} := \dfrac{r_B}{\sqrt{2}\cdot sin(\beta'_0)} \quad r_{t'} = 0.827\,r_B \quad x_{0t}(\alpha_t) := -r_B + r_{t'}\cdot(1 + cos(\alpha_t)) \quad y_{0t}(\alpha_t) := r_{t'}\cdot sin(\alpha_t)$

$x_{0t'}(\alpha_{t'}) := \left[r_B + r_{t'}\cdot(-1 + cos(\alpha_{t'}))\right]cos(\alpha_B) - r_{t'}\cdot sin(\alpha_{t'})\cdot sin(\alpha_B) \quad l'_t := r_{t'}\cdot 2\cdot\beta'_0$

$y_{0t'}(\alpha_{t'}) := \left[r_B + r_{t'}\cdot(-1 + cos(\alpha_{t'}))\right]sin(\alpha_B) + r_{t'}\cdot sin(\alpha_{t'})\cdot cos(\alpha_B) \quad L_t := l_t + L + l'_t$

Position des goupilles de raquettes $\quad r_{GR} := r_{t2} \quad \alpha_{GR} := -\beta_0 \quad \alpha_{GR} = -82.695\,deg$

$x_{GR} := x_{0t2}(\alpha_{GR}) \quad y_{GR} := y_{0t2}(\alpha_{GR})$

Position du point d'attache à la virole $\quad r_V := \sqrt{x_{0t'}(2\cdot\beta'_0)^2 + y_{0t'}(2\cdot\beta'_0)^2} \quad \alpha_V(\theta) := Atan\left(x_{0t'}(2\cdot\beta'_0), y_{0t'}(2\cdot\beta'_0)\right) + \theta$

$r_V = 0.55\,mm \quad \alpha_V(0) = 216.447\,deg \quad x_V(\theta) := r_V\cdot cos(\alpha_V(\theta)) \quad y_V(\theta) := r_V\cdot sin(\alpha_V(\theta))$

Amplitude stationnaire du balancier $\quad \theta_0 = 270\,deg$

Contrainte maximum

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$ $W_{f3} := W_{f\_rect}(ép, ha)$ $\sigma_{max} := \dfrac{E \cdot I_{33}}{L \cdot W_{f3}} \cdot \theta_0$ $\sigma_{max} = 138.587\, N \cdot mm^{-2}$

Centres de masse

Partie spiralée

$z_{0s}(\alpha) := x_{0s}(\alpha) + i \cdot y_{0s}(\alpha)$

$\zeta_{0s} := \dfrac{1}{L} \cdot \int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha) \cdot r_s(\alpha)\, d\alpha$ $\xi_{0s} := Re(\zeta_{0s})$ $\eta_{0s} := Im(\zeta_{0s})$

$\xi_{0s} = -4.111 \times 10^{-3}\, mm$ $\eta_{0s} = -0.052\, mm$

Courbe terminale externe

$z_{0t1}(\alpha_t) := x_{0t1}(\alpha_t) + i \cdot y_{0t1}(\alpha_t)$ $z_{0t2}(\beta_t) := x_{0t2}(\beta_t) + i \cdot y_{0t2}(\beta_t)$

$$\zeta_{0t} := \frac{1}{l_t} \cdot \left(\int_0^{\pi} z_{0t1}(\alpha_t) \cdot r_{t1}\, d\alpha_t + \int_{-\beta_0}^{0} z_{0t2}(\beta_t) \cdot r_{t2}\, d\beta_t \right)$$

$\xi_{0t} := Re(\zeta_{0t})$ $\eta_{0t} := Im(\zeta_{0t})$ $\xi_{0t} = 0\, mm$ $\eta_{0t} = 0.632\, mm$

Courbe terminale interne

$z_{0t'}(\alpha_{t'}) := x_{0t'}(\alpha_{t'}) + i \cdot y_{0t'}(\alpha_{t'})$

$\zeta_{0t'} := \dfrac{r_{t'}}{l'_t} \cdot \int_0^{2 \cdot \beta'_0} z_{0t'}(\alpha_{t'})\, d\alpha_{t'}$ $\xi_{0t'} := Re(\zeta_{0t'})$ $\eta_{0t'} := Im(\zeta_{0t'})$ $\xi_{0t'} = 0.126\, mm$ $\eta_{0t'} = 0.163\, mm$

Centre de masse du spiral

$\zeta_s := \dfrac{1}{L_t} \cdot (L \cdot \zeta_{0s} + l_t \cdot \zeta_{0t} + l'_t \cdot \zeta_{0t'})$ $\zeta_s = -1.032 \times 10^{-3} + 1.753i \times 10^{-4}\, mm$

Première approximation de la déformée du spiral

Courbe terminale externe

$\varphi_{0t2}(\beta_t) := \dfrac{\pi}{2} + \beta_t$ $z_{GR} := x_{GR} + i \cdot y_{GR}$ $z_{1t2}(\theta, \beta_t) := z_{GR} + r_{t2} \cdot \displaystyle\int_{-\beta_0}^{\beta_t} i \cdot e^{i \cdot \beta'_t} \cdot exp\left[i \cdot \frac{\theta}{L_t} \cdot \left[r_{t2} \cdot (\beta_0 + \beta'_t) \right] \right] d\beta'_t$

$$z_{1t2}(\theta, \beta_t) := z_{GR} + \frac{L_t \cdot r_{t2}}{L_t + \theta \cdot r_{t2}} \cdot \left[exp\left[i \cdot \frac{\beta_t \cdot L_t + \theta \cdot r_{t2} \cdot (\beta_0 + \beta_t)}{L_t} \right] - exp(-i \cdot \beta_0) \right]$$

$z_{1C}(\theta) := z_{1t2}(\theta, 0)$

$\Delta\varphi_{1C}(\theta) := \dfrac{\theta}{L_t} \cdot r_{t2} \cdot \beta_0$ $\Delta\varphi_{1C}(\theta_0) = 4.989\, deg$

$\varphi_{0t1}(\alpha_t) := \alpha_t + \dfrac{\pi}{2}$ $\Delta z_{1t1}(\theta, \alpha_t) := r_{t1} \cdot \displaystyle\int_0^{\alpha_t} i \cdot e^{i \cdot \alpha'_t} \cdot exp\left(i \cdot \frac{\theta}{L_t} \cdot r_{t1} \cdot \alpha'_t \right) d\alpha'_t$

$\Delta z_{1t1}(\theta, \alpha_t) := \dfrac{L_t \cdot r_{t1}}{L_t + \theta \cdot r_{t1}} \cdot \left(exp\left(i \cdot \alpha_t \cdot \dfrac{L_t + \theta \cdot r_{t1}}{L_t} \right) - 1 \right)$ $z_{1t1}(\theta, \alpha_t) := z_{1C}(\theta) + \Delta z_{1t1}(\theta, \alpha_t) \cdot e^{i \cdot (\Delta\varphi_{1C}(\theta))}$

$\Delta\varphi_{1A}(\theta) := \theta \cdot \dfrac{l_t}{L_t}$ $\Delta\varphi_{1A}(\theta_0) = 18.587\, deg$ $z_{1A}(\theta) := z_{1t1}(\theta, \pi)$

Partie spiralée

$s'(\alpha) := r_A - a\cdot(\alpha - \alpha_A)$ $\qquad z'_0(\alpha) := \left[-a + i\cdot\left[r_A - a\cdot(\alpha - \alpha_A)\right]\right]\cdot exp(i\cdot\alpha)$

$\Delta z_{1s}(\theta,\alpha) := \int_{\alpha_A}^{\alpha} z'_0(\alpha')\cdot exp\left(i\cdot\theta\cdot\frac{s(\alpha')}{L_t}\right) d\alpha'$ $\qquad z_{1s}(\theta,\alpha) := z_{1A}(\theta) + \Delta z_{1s}(\theta,\alpha)\cdot e^{i\cdot\Delta\varphi_{1A}(\theta)}$

Courbe terminale interne

$\Delta z_{1t'}(\theta,\alpha_{t'}) := r_{t'}\cdot\int_0^{\alpha_{t'}} i\cdot exp(i\cdot\alpha'_{t'})\cdot exp\left(i\cdot\theta\cdot\frac{r_{t'}}{L_t}\cdot\alpha'_{t'}\right) d\alpha'_{t'}$

$\Delta z_{1t'}(\theta,\alpha_{t'}) := \frac{r_{t'}\cdot L_t}{\theta\cdot r_{t'} + L_t}\cdot\left(exp\left(i\cdot\alpha_{t'}\cdot\frac{\theta\cdot r_{t'} + L_t}{L_t}\right) - 1\right)$ $\qquad z_{1B}(\theta) := z_{1s}(\theta, \psi_0 + \pi)$ $\qquad \alpha_B = 322.4\,deg$

$\alpha_{1B}(\theta) := Atan\left(Re(z_{1B}(\theta)), Im(z_{1B}(\theta))\right)$ $\qquad z_{1t'}(\theta,\alpha_t) := z_{1B}(\theta) + \Delta z_{1t'}(\theta,\alpha_t)\cdot e^{i\cdot\alpha_{1B}(\theta)}$

Graphe de la déformation

Forme naturelle

$n_t := 201$ $\quad j := 0..n_t - 1$ $\quad \Delta\alpha_t := \frac{\pi}{n_t - 1}$ $\quad \alpha_{t_j} := j\cdot\Delta\alpha_t$ $\quad x_{t1_j} := x_{0t1}(\alpha_{t_j})$ $\quad y_{t1_j} := y_{0t1}(\alpha_{t_j})$

$\Delta\beta_t := \frac{\beta_0}{n_t - 1}$ $\quad \beta_{t_j} := j\cdot\Delta\beta_t - \beta_0$ $\quad x_{t2_j} := x_{0t2}(\beta_{t_j})$ $\quad y_{t2_j} := y_{0t2}(\beta_{t_j})$ $\quad x_t := pile(x_{t2}, x_{t1})$ $\quad y_t := pile(y_{t2}, y_{t1})$

$n := 50\cdot partentière(n_{sp}) + 1$ $\quad i := 0..n - 1$ $\quad \Delta\alpha := \frac{\psi_0}{n - 1}$ $\quad \alpha_i := \pi + i\cdot\Delta\alpha$

$x_{s_i} := x_{0s}(\alpha_i)$ $\quad y_{s_i} := y_{0s}(\alpha_i)$ $\quad x_0 := pile(x_t, x_s)$ $\quad y_0 := pile(y_t, y_s)$ $\quad mod(\psi_0 + \pi, 2\cdot\pi) = 322.4\,deg$

$\Delta\alpha_{t'} := \frac{2\cdot\beta'_0}{n_t - 1}$ $\quad \alpha_{t'_j} := j\cdot\Delta\alpha_{t'}$ $\quad x_{t'1_j} := x_{0t'}(\alpha_{t'_j})$ $\quad y_{t'1_j} := y_{0t'}(\alpha_{t'_j})$ $\quad x_0 := pile(x_0, x_{t'1})$ $\quad y_0 := pile(y_0, y_{t'1})$

$r_0 := \overrightarrow{\sqrt{x_0^{\,2} + y_0^{\,2}}}$ $\quad \beta_s := \overrightarrow{Atan(x_0, y_0)}$ $\quad n_{pt} := dernier(\beta_s)$ $\quad \beta_{s_{npt}} = 216.4\,deg$ $\quad \alpha_V(0) = 216.4\,deg$

Déformée

$z_{td2} := \overrightarrow{z_{1t2}(\theta_0, \beta_t)}$ $\quad z_d := z_{td2}$ $\quad z_{td1} := \overrightarrow{z_{1t1}(\theta_0, \alpha_t)}$ $\quad z_d := pile(z_{td2}, z_{td1})$

$z_{sd} := \overrightarrow{z_{1s}(\theta_0, \alpha)}$ $\quad z_d := pile(z_d, z_{sd})$ $\quad z_{t'd} := \overrightarrow{z_{1t'}(\theta_0, \alpha_{t'})}$ $\quad z_d := pile(z_d, z_{t'd})$

$n_{pt} := dernier(z_d)$ $\quad x_d := Re(z_d)$ $\quad y_d := Im(z_d)$ $\quad r_d := \overrightarrow{(|z_d|)}$ $\quad r_{d_{npt}} = 0.547\,mm$

$\beta_d := \overrightarrow{Atan(x_d, y_d)}$ $\quad \beta_{d_0} = 277.305\,deg$ $\quad \beta_{d_{npt}} = 127.682\,deg$ $\quad mod(\alpha_V(\theta_0), 2\cdot\pi) = 126.447\,deg$

$r_{GR} = 1.502\,mm$ $\quad r_V = 0.55\,mm$ $\quad x_V(\theta_0) = -0.327\,mm$ $\quad y_V(\theta_0) = 0.442\,mm$

$x_{d_{npt}} - x_V(\theta_0) = -7.737\times 10^{-3}\,mm$ $\qquad y_{d_{npt}} - y_V(\theta_0) = -9.376\times 10^{-3}\,mm$

Spiral plat
avec courbes terminales